Measurable and Immeasurable in Relativistic Quantum Information

MSc Dissertation

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Abstract

The field of relativistic quantum information (RQI) has emerged from studying quantum information (QI) in a relativistic context. Since its inception it has become useful as a means of tackling problems ranging from QI processing techniques to black hole physics. The increased use of quantum fields in RQI research has necessitated a better understanding of measurement in QFT. An analysis of possible operators in QFT demonstrates that causality limits the number of possible quantum operations, arriving at definitions of operations which are causal, acausal or semicausal, and localisable, semilocalisable and nonlocal. Then an example is given where an idealised measurement, generalised from nonrelativistic quantum mechanics to QFT, produces superluminal signalling. Furthermore this problem persists even with arbitrarily long measurement times on spatially localised Gaussian-like wavepackets. Finally arguments are presented for strategies to avoid such unphysical behaviour before concluding that the path integral formulation of quantum mechanics is the most suitable for working with relativistic quantum field theory.

Contents

Introduction

The link between physics and information can be traced back to the origins of statistical mechanics. Searching for the correct mathematical definition of entropy and building on work by Boltzmann and others, Planck arrived at the equation, $S = k \log W$, in 1901 [1]. This now familiar result states that the entropy of a system is logarithmically related to the number of microstates corresponding to the same macrostate. As the microstates specify the possible positions of particles in the system, one can see that entropy is a form of information. Contemporary investigations discovered that if the probability of the *i'th* microstate of some system is p_i then the entropy is, $S = k_B(-\sum_i p_i \ln p_i)$. The term, $\sum_i p_i \ln p_i$, has since appeared in similar form in the seminal paper on information theory as the Shannon entropy which quantifies the expected value of information of a message [2]. It also appears in the von Neumann entropy in that formulation of quantum mechanics [3].

In order for S to be finite there can only be a finite number of microstates (without some regularisation scheme). For classical physics this is problematic as a particle may inhabit an infinite array of positions. One consequence of this is the prediction that a black body radiates an infinite amount of energy. Planck famously solved this problem by proposing that energy is discrete and proportional to the Planck constant, h [1]. In this first paper on quantum theory entropy written as, $S = k \log W$, also appears for the first time so information and quantum mechanics have had a long association.

Quantum information as an independent research area gained prominence in the 1990's inspired by earlier work in applying quantum mechanics to communication and computation. In the 60's J. P. Gordon [4] and Lev Levitin [5] conducted analyses of capacity in classical communication channels while in 1973 Holevo derived the capacity for quantum mechanical channels to transmit classical information [6]. This lead to studies of the properties of quantum information channels [7], [8] and the use of teleportation as a means of transmitting quantum information [9]. Teleportation has since been demonstrated experimentally for photons and atoms [10], [11]. Quantum computing has its origins in Benioff's 1980 study that showed quantum mechanical systems could perform classical computations [12]. In 1982 Feynmann postulated that quantum systems would be better simulated using quantum mechanical simulators [13] and in 1985 Deutsch developed the first model of a quantum Turing machine [14]. Without practical applications this research remained a curiosity until Shor's algorithm demonstrated that a quantum computer could efficiently factor large numbers [15]. Since the difficulty of number factorisation ensures the security of public key encryption protocols research into quantum computation acquired much greater interest. Subsequently other useful quantum algorithms have been found [16] and prototype quantum computers and logic gates have been built [17], [18]. While quantum computers have the potential to break contemporary codes, quantum mechanics provides a solution to secure communication. In 1984 Bennett and Brassard [19] developed a form of quantum cryptography which cannot be intercepted without the knowledge of the communicating parties. Working independently Ekert demonstrated another encryption protocol based on entanglement [20]. Quantum cryptography is now becoming a technical reality.

The themes of computation, communication and cryptography form the main focus of the quantum information research effort. Absent from the discussion of quantum information thus far is the role of relativity. Much progress has been made by ignoring relativistic effects in quantum systems but in certain instances it cannot be overlooked, a clear example being certain quantum information processing schemes involving photons. More generally relativity is in essence concerned with the exchange of signals between observers in relative motion. Therefore, fundamentally it is a theory of information transfer and as such any generally consistent quantum information theory must include relativistic effects.

The first treatment of quantum information in relativistic terms was by Czachor who demonstrated that relativistic effects influence the Einstein-Rosen-Podolsky-Bohm experiment where the degree of violation of Bells' inequalities depends on the velocity of the entangled particles [21]. Another investigation by Peres and Terno [22], shows that for a spin 1/2 system, spin entropy is not a Lorentz covariant quantity. Further notable results show that although entanglement may be preserved in inertial reference frames, spin and position degrees of freedom may change [23]. For noninertial frames it has been shown that entanglement is observer dependent [24–26]. Treating quantum information relativistically has also been shown to be a fruitful approach in dealing with other areas of physics such as the information loss problem in black holes [27] and also in studies of quantum gravity [28].

More recent activity in relativistic quantum information demonstrates the increasing use of quantum fields. Martin-Martinez *et al.* [29] present a scheme for a quantum thermometer which couples a qubit to quantum fields through an atomic interferometer, using the resulting Berry phase to ascertain temperature. A similar idea appears in [30] for a weak measurement process to detect photons without significant disturbance. Another study by Martin-Martinez *et al.* [31] proposes a quantum gate controlled by the relativistic motion of a qubit coupled to a quantum field in a cavity. Martin-Martinez *et al.* [32] demonstrate a protocol for creating entangled particles from a quantum field by sending unentangled particles through an optical cavity while

Doukas *et al.* [33] show the influence of noninertial motion on entanglement by coupling an Unruh detector to a mode of a quantum field.

Of utmost importance in any quantum information processing scheme is the ability to read the result of the process. This clearly requires making some kind of measurement by an externally acting agent. Given the growing prevalence of quantum field interactions within the relativistic quantum information research oeuvre it is pertinent to assess what kinds of measurements can be made.

Chapter 1

Allowable Quantum Operations in Relativistic Quantum Theory

Given the axioms of quantum mechanics and its associated mathematical structure one might suppose that any self adjoint operator defined on a spacelike hypersurface should be measurable. Beckman *et al.* [34] consider the effects of relativistic causation on such operators and the caveats they impose. In the discussion they consider a quantum system divided into two parts A and B presided over respectively by the archetypal quantum information protagonists Alice and Bob. Initially Alice and Bob share the density operator of the whole system ρ, together with some ancilla state or states and are spacelike separated so that communication is forbidden. Alice and Bob will transform the density operator ρ using some particular operation.

In general an operation which can effect allowable changes in a quantum system is classed as a linear map between density operators that is completely positive and trace nonincreasing. An important subset of such operations are trace preserving superoperators $\mathcal{E}$, whose actions are defined on a density operator ρ as,

$$\mathcal{E}(\rho) = \sum_{\mu} M_{\mu} \rho M_{\mu}^{\dagger} \tag{1.1}$$

where M_{μ} are normalised such that,

$$\sum_{\mu} M_{\mu} M_{\mu}^{\dagger} = I \tag{1.2}$$

$\mathcal{E}$ is interpreted as a generalised measurement with an unknown outcome. If the outcome has been previously chosen but the density operator is not renormalised this is classed as an operation. A generalised operation can be cast in the same form as 1.1 by summing μ over a restricted set of operators obeying 1.2 such that the eigenvalues of $\sum_{\mu} M_{\mu}^{\dagger} M_{\mu}$ are no greater than 1. In general the probability of an observed outcome of an operation is given by $tr\mathcal{E}(\rho)$.

The superoperator $\mathcal{E}_S$ acting on Hilbert space $\mathcal{H}_S$ is enacted by using an ancilla with Hilbert space $\mathcal{H}_R$. A pure state $\mid \psi\rangle \in \mathcal{H}_R$ from the ancilla is prepared then a unitary transformation U, unique to that operation, is applied to $\mathcal{H}_S \otimes \mathcal{H}_R$, and the ancilla state discarded,

$$\mathcal{E}_S(\rho_S) = tr_R[U(\rho_S \otimes \mid \psi\rangle_{RR}\langle\psi \mid)U^{\dagger}] \tag{1.3}$$

A general operation is implemented similarly with the addition of an orthogonal measurement on the ancilla with a particular result chosen after U is applied.

Having defined superoperators and their actions it is pertinent to consider which of the broad range of superoperators are physically possible. It has been shown [35–39] that relativistic causality forbids the implementation of large classes of such operators conforming to 1.1. For instance suppose Bob performed a local operation before implementing $\mathcal{E}$ then Alice performed her own local operation thereafter. $\mathcal{E}$ would be forbidden if Alice were able to learn about Bob's operation as this would be a superluminal signal. More specifically consider the operation $\mathcal{E}_{AB}$ acting on Alice and Bob's combined Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$. Alice is allowed to control an ancilla state R and Bob an ancilla state S. Using the operator $\mathcal{E}$ Bob can attempt to signal Alice superluminally. An initial shared density operator ρ_{RABS} corresponding to $\mathcal{H}_R \otimes \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_S$ can be operated upon by Bob using a local superoperator $\mathcal{B}_S$ so that its trace preserving nature does not alert Alice to the outcome of Bob's measurement. After the application of $\mathcal{E}$ Alice's density operator is obtained by tracing over Bob's system and ancilla,

$$\rho_{RA} = tr_{BS}[\mathcal{E}_{AB}(I_{RA} \otimes \mathcal{B}_{BS}(\rho_{RABS}))] \tag{1.4}$$

where $\mathcal{E}_{AB}$ acts trivially on the ancilla states. In the situation that $\mathcal{E}$ is not trace preserving ρ_{RA} will not be normalised therefore Alice's final state is the normalised density operator $\rho_{RA}/tr\rho_{RA}$. If the measurement that Alice makes on this final state depends on the superoperator $\mathcal{B}$ Bob can superluminally signal to Alice. For an operation that forbids such one way communication (from Bob to Alice) that operator is called semicausal. Hence the formal definition for a semicausal operator is then:

A bipartite operation $\mathcal{E}$ is semicausal iff $(\rho_{RA}/tr\rho_{RA})$ is independent of $\mathcal{B}$ for all shared initial states ρ_{RABS}.

For an operator that is semicausal in both directions i.e. where neither party can signal each other this is defined as a causal operator.

The definition for semicausality may be simplified, without loss of generality, by allowing the shared initial state ρ_{RABS} to be a product state rather than an entangled state. If a superoperator is not semicausal there is a local operator $\mathcal{B}$ that Bob can apply to ρ_{RABS} such that,

$$tr_{BS}[\mathcal{E}_{AB}((\rho_{RABS}))] \neq tr_{BS}[\mathcal{E}_{AB}(I_{RA} \otimes \mathcal{B}_{BS}(\rho_{RABS}))] \tag{1.5}$$

In general one may expand any density operator as,

$$\rho_{RABS} = \sum_{\mu} \lambda_{\mu} \rho_{\mu} \otimes \sigma_{\mu} \tag{1.6}$$

where ρ_{μ} and σ_{μ} are Alice and Bob's respective density operators. Therefore eqn. 1.5 may be rewritten as,

$$\sum_{\mu} \lambda_{\mu} tr_{BS}[\mathcal{E}(\rho_{\mu} \otimes \sigma_{\mu})] \neq \sum_{\mu} \lambda_{\mu} tr_{BS}[\mathcal{E}(\rho_{\mu} \otimes \mathcal{B}(\sigma_{\mu}))] \tag{1.7}$$

which is only true when,

$$tr_{BS}[\mathcal{E}(\rho_{\mu} \otimes \sigma_{\mu})] \neq tr_{BS}[\mathcal{E}(\rho_{\mu} \otimes \mathcal{B}(\sigma_{\mu}))] \tag{1.8}$$

for at least one value of μ. Thus for the product state $\rho_{\mu} \otimes \sigma_{\mu}$, $\mathcal{E}$ allows Bob to signal Alice superluminally. The density operators ρ_{μ} and σ_{μ} are ensemble pure states so there exists a signalling regime in which Alice and Bob's initial states are pure. Therefore Bob can signal Alice by choosing between pure states $\mid \psi\rangle_{BS}$ and $\mid \psi'\rangle_{BS}$. Since tracing over $\mathcal{S}$ commutes with $\mathcal{E}$ Bob may equally begin with mixed state ρ_B, ρ'_B of B. In that instance signalling Alice requires Bob prepare pure states of the concomitant density operators. Alice can receive signals by preparing an initial pure state $\mid \varphi\rangle_{RA}$. System A is then susceptible to two final states influenced by Bob's actions and $\mathcal{E}$ which produces different outcomes when acting on one of Alice's pure states RA. Consequently Alice may dispense with the R ancilla and work with A alone. Thus one may say that if a superoperator $\mathcal{E}$ is not semicausal, signalling can occur with pure initial states minus ancillas: for $\mid \psi'\rangle_B$, $\mid \psi\rangle_B \in \mathcal{H}_B$ and $\mid \varphi\rangle_A \in \mathcal{H}_A$,

$$\begin{aligned} &tr_B[\mathcal{E}((\mid \varphi\rangle\langle\varphi \mid)_A \otimes (\mid \psi\rangle\langle\psi \mid)_B)] \\ &\quad \neq tr_B[\mathcal{E}((\mid \varphi\rangle\langle\varphi \mid)_A \otimes (\mid \psi'\rangle\langle\psi' \mid)_B)] \end{aligned} \tag{1.9}$$

Due to the definition and linearity of semicausal superoperators one can see that they form a convex set such that for superoperator $\mathcal{E}_a$ the sum,

$$\mathcal{E} = \sum_a p_a \mathcal{E}_a \tag{1.10}$$

is also a superoperator, where p_a is nonnegative and $\sum_a p_a = 1$. Another property of semicausal operators is that they form a semi group such that $\mathcal{E}_1 \circ \mathcal{E}_2$ is also semicausal.

As Alice and Bob are spacelike separated and therefore cannot communicate any operation they choose to carry out must be implemented locally. An operation that can be implemented thus is called localisable. If Alice and Bob are allowed to share some possibly entangled ancilla state previous to the operation they wish to enact one can make a formal definition of a localisable operation.

A bipartite superoperator $\mathcal{E}$ is localisable iff for shared ancilla state ρ_{RS} and local superoperators $\mathcal{A}_{RA}$ and $\mathcal{B}_{BS}$,

$$\mathcal{E}(\rho_{AB}) = tr_{RS}[\mathcal{A}_{RA} \otimes \mathcal{B}_{BS}(\rho_{AB} \otimes \rho_{RS})] \tag{1.11}$$

By replacing the local superoperators with unitary transformations and extending the ancilla state so that ρ_{RS} becomes an ensemble of pure states one may redefine localisability as,

$$\mathcal{E}(\rho_{AB}) = tr_{RS}[U_{RA} \otimes V_{BS}(\rho_{AB} \otimes \rho_{RS})U_{RA}^{\dagger} \otimes V_{BS}^{\dagger}] \tag{1.12}$$

Like semicausal operators localisable superoperators also form a convex set. The shared entanglement of Alice and Bob's ancilla state allows them to simulate shared randomness. For an example ancilla state,

$$\mid \phi\rangle_{RS} = \sum_a \sqrt{p_a} \mid a\rangle_R \otimes \mid a\rangle_S \tag{1.13}$$

where $| a\rangle_R$ form an orthonormal basis in Alice's Hilbert space $\mathcal{H}_R$ and $| a\rangle_S$ is the orthonormal basis in Bob's Hilbert space $\mathcal{H}_S$ and where $\sum_a p_a = 1$. If Alice and Bob perform measurements on these states they each obtain the result $| a\rangle$ with probability p_a. For a localisable set of operators $\mathcal{E}_a$ Alice and Bob can consult their shared randomness and then apply $\mathcal{E}_a$ with probability p_a resulting in the convex sum $\sum_a p_a \mathcal{E}_a$. A tensor product $\mathcal{E}_A \otimes \mathcal{E}_B$ of localisable superoperators constructed by Alice and Bob is also a superoperator and by applying this convexity property any superoperator of the form,

$$\mathcal{E} = \sum_a p_a \mathcal{E}_{A,a} \otimes | a\mathcal{E}_{B,a} \tag{1.14}$$

is localisable as well.

Of equal interest are the set of operators which are semilocalisable where, like semicausality, communication exists in just one direction. Under the condition of semilocalisability if Alice wishes to message Bob he must be in Alice's forward light cone. Alice can then send Bob qubits to establish shared entanglement allowing classical communication. Equivalently they can share a prior entangled state allowing Alice to teleport information to Bob. As a result Alice and Bob can share an ancilla state. With access to the same ancilla Alice can perform a local operation on the ancilla and her half of the shared state, which she then sends to Bob who performs his own local operation likewise. Thus for some ancilla state ρ_R, one can define semilocalisability as,
A bipartite operation $\mathcal{E}$ is semilocalisable iff

$$\mathcal{E}(\rho_{AB}) = tr_R[(\mathcal{B}_{BR} \circ \mathcal{A}_{RA})(\rho_{AB} \otimes \rho_R)] \tag{1.15}$$

where $\mathcal{A}_{RA}$ is an operation and $\mathcal{B}_{BR}$ is a superoperator. Also $\mathcal{B}_{BR} \circ \mathcal{A}_{RA}$ is a composition rather than a tensor product as the operations that Bob and

Alice perform do not commute because they act on the same ancilla.

In this construct Alice may act locally with a nontrace preserving operator as Bob is allowed to know her measurement outcome. Bob however, is restricted to a superoperator as Alice may not know about Bob's measurement. If $\mathcal{E}$ is a superoperator $\mathcal{A}_{RA}$ must also be therefore both $\mathcal{A}_{RA}$ and $\mathcal{B}_{BR}$ can be taken as unitary transformations. A corollary of the definition of semilocalisablity is that semilocalisable and localisable operators form a semigroup such that if $\mathcal{E}_1$ and $\mathcal{E}_2$ are semilocalisable then $\mathcal{E}_1 \circ \mathcal{E}_2$ is semilocalisable also.

To illustrate these concepts one may take the example, attributable to Sorkin [39], of a two outcome Bell measurement on a pair of qubits. Taking the state $|\phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ the orthogonal projectors corresponding to the two outcomes are,

$$\begin{aligned} E_1 &= |\phi^+\rangle\langle\phi^+|, \\ E_2 &= I - |\phi^+\rangle\langle\phi^+| \end{aligned} \tag{1.16}$$

Initially Bob and Alice may share a state $|01\rangle_{AB}$, which is orthogonal to $|\phi^+\rangle$. In this case outcome two will always occur as it is orthogonal to $|\phi^+\rangle$, leaving Alice with density operator $\rho_A = |0\rangle\langle 0|$. However if Bob uses a unitary transformation on $|01\rangle_{AB}$ to rotate it into the state $|00\rangle_{AB}$ this will result in either outcome occurring with equal probability. For both outcomes the final state is maximally entangled giving Alice a density operator of $(1/2)I$. Thus she can distinguish between the case where Bob has applied his local operation and when he has not, therefore operators 1.16 are acausal.

Not all orthogonal measurement superoperators are acausal however. A measurement on the tensor product $A \otimes B$ is causal as Alice and Bob can induce decoherence in its basis eigenstates using local operations. Another example is complete Bell measurement, which in fact are the only nonlocal measurements allowed on two qubit systems in relativistic quantum mechanics [38]. For the Bell states,

$$
\begin{aligned}
|\,\phi^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle \pm \langle 11|), \\
|\,\psi^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|01\rangle \pm \langle 10|)
\end{aligned}
\tag{1.17}
$$

Bob can in no way influence the shared state, which is always maximally entangled. Therefore Alice will always have a density operator $\rho_B = I/2$ and won't be able to discern any of Bob's previous actions.

A further example demonstrates the distinction between localisable and semilocalisable causal operators. Consider the two step superoperator $\mathcal{E}_{\wedge}$. If Alice and Bob share a two qubit state, step one of the operation is an orthogonal projection on to the product basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. Step two consists of transforming the basis thus,

$$
\begin{aligned}
\left.\begin{array}{l} |00\rangle \\ |01\rangle \\ |10\rangle \end{array}\right\} &\rightarrow \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|) \\
|11\rangle &\rightarrow \frac{1}{2}(|01\rangle\langle 01| + |10\rangle\langle 10|)
\end{aligned}
\tag{1.18}
$$

This operation is both trace preserving and completely positive and is also causal as the final density operator is $\rho = I/2$ regardless of the shared state Alice and Bob begin with. However, $\mathcal{E}_{\wedge}$ is not localisable as Alice and Bob must communicate to enact it.

To see this let the shared input state be chosen from $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. $\mathcal{E}_{\wedge}$ is implemented by applying a local unitary transformation and the output state is measured in the $\{|0\rangle, |1\rangle\}$ basis. Explicitly Alice applies $U_A^{-1} Z_{A,out} U_A$ to the initial state while Bob uses $U_B^{-1} Z_{B,out} U_B$, where $Z_{.,out}$ represents a Pauli operator acting on the output. The observables measured in this process have eigenvalues ± 1. This situation can be modified to one in which Alice and Bob are measuring one of two fixed observables A, A' and B, B' respectively. One can then apply the Csirel'son inequality,

$$\langle AB \rangle + \langle AB' \rangle + \langle A'B \rangle - \langle A'B' \rangle \leq 2\sqrt{2} \tag{1.19}$$

to relate Alice and Bob's outcomes. For instance, instead of $|0\rangle$ or $|1\rangle$ Alice can start with the state $|0\rangle$ and then, before her measurement is made, she can either apply the Pauli operator X to it or not. This amounts to Alice receiving a classical input bit instructing her to measure either of the observables,

$$\begin{aligned} A &= U_A^{-1} Z_{A,out} U_A \\ A' &= X_{A,in} U_A^{-1} Z_{A,out} U_A X_{A,in} \end{aligned} \tag{1.20}$$

and likewise for Bob.

1.19 then determines how Alice's measurement $|a\rangle$ is correlated with Bob's measurement $|b\rangle$. For Alice and Bob to have successfully applied $\mathcal{E}_\wedge$ the outcomes are related to the classical input bits x and y as $a \oplus b = x \wedge y$ with probability 1. This contradicts the Cirel'son inequality and therefore $\mathcal{E}_\wedge$ cannot be locally enacted.

However $\mathcal{E}_\wedge$ is semilocalisable as it can be enacted using one way classical communication. If Alice measures a qubit in the $\{|0\rangle, |1\rangle\}$ basis she can then toss a coin to determine whether to flip the state. Afterwards she sends both the outcome of the coin toss and her measurement to Bob. Bob measures his qubit in the same basis then using the information received from Alice flips his qubit to match Alice's unless they both measure $|1\rangle$ in which case he gives his qubit the opposite value. Thus the operator $\mathcal{E}_\wedge$ has been successfully employed.

Chapter 2

Idealised Measurements in QFT

The previous chapter focused on categorising the kinds of operation that are possible in a relativistic quantum theory. A specific example of a measurement is provided in [39]. In this paper Sorkin defines a causal structure for spacetime which is used to assign a linear order to observables. This allows for a generalisation of idealised measurement in quantum theory to quantum field theory. It is then shown that measurements of this kind ultimately lead to superluminal signalling.

An idealised measurement fulfills two criteria. First, the eigenvalues corresponding to a particular operator represent the possible outcomes of the measurement with probabilities governed by the conventional trace rule. Second, the effect of the measurement on the subsequent quantum state is correctly described by the projection postulate. This example of a 'minimally disturbing measurement' represents simultaneous detection and preparation of a state. In nonrelativistic quantum mechanics an idealised measurement is considered to occur at a specific moment in time. The equivalent procedure in quantum field theory assumes that a measurement takes place on a Cauchy hypersurface. In essence a Cauchy hypersurface is the spacetime analogue of an instant in time consisting of a set of spacelike separated points intersected by any timelike curve. To allow for the use of well defined operators the hypersurface is thickened, an idea compatible with the most general quantum field theories where it is assumed that an algebra of observables exist within a spacetime region which are measurable entirely within that region.

Assuming that an observable A is associated to a particular spacetime region O it is possible to incorporate a generalised projection postulate into the definition of such a measurement. Difficulties arise however if one wishes to make more than one measurement. In nonrelativistic quantum mechanics there is a specific time order to measurement events but in a relativistic setting event order is less obvious. Therefore it becomes necessary to establish a framework within which to properly generalise an idealised measurement to quantum field theory. For simplicity one can adopt Minkowski space without loss of generality regarding extension to other globally hyperbolic spacetime geometries. One may also work in the Heisenberg picture as this lends itself more appropriately to the association of field operators with specific regions of spacetime.

Idealised measurements are made on some quantum field Φ. A set of regions O_k inhabiting Minkowski space have assigned to them an observable A_k resulting from making a measurement on Φ confined to that region. With an initial state ρ_0 occurring to the past of all O_k, one can ascertain the probability of finding the eigenvalues α_k of A_k as outcomes of a particular measurement. In nonrelativistic quantum mechanics these probabilities can be given a time ordering such that A_1 precedes A_2 precedes A_3 etc. Probabilities are calculated using ρ_0 for the earliest observable A_1 then ρ is reduced using the condition of α_1 and used to determine the probability for A_2 and so on. In the special case of A_k acting as a projector E_k this procedure results in,

$$\langle E_1 E_2 ... E_{n-1} E_n 1 E_{n+1} ... E_2 E_1 \rangle \tag{2.1}$$

In this equation and throughout Sorkin uses angle bracket notation such that $\langle A \rangle \equiv tr(\rho_0 A))$.

For the special case where O_k are nonintersecting Cauchy hypersurfaces this analysis can be directly transported to the relativistic situation as unambiguous time ordering is guaranteed. In a more general setting it may be possible to induce a well defined time ordering of O_k by foliating the spacetime.

Using the symbol $\prec$ one may assign labellings that reflect the causal relationships between various regions. The relation $\prec$ is defined as $O_j \prec O_k$ iff a point in O_j causally precedes some point in O_k. A linear ordering of regions is then consistent with $\prec$ iff $O_j \prec O_k$ which implies that $j \leq k$. It might possible that one cannot label regions in such a manner forbidding generalisation of the above probability rules represented in 2.1. To exclude the possibility that such labellings don't exist the transitive closure of $\prec$ is taken. If $O_j \prec O_k$ and $O_k \prec O_j$ implies $j = k$ the $\prec$ is a partial order. In this case the regions always admit a linear order such that labels $i = 1, ..., n$ for regions $O_j \prec O_k$ imply $j \leq k$. This is assumed to be the case in the following.

With this definition in place one may naturally generalise the probability rules stated above by extending $\prec$ to a linear order. The choice of ordering may not be unique but is not a problem while Φ obeys local commutativity such that observables in spacelike separated regions commute. When all A_k are projection operators 2.1 is valid for all labelling schemes of regions compatible with the partial $\prec$. However, there is a problem with this scheme which relates to the transitive closure. When taking a measurement the resulting state vector reduction implies nonlocality allowing observable effects to be transmitted superluminally. Therefore, to obviate the possibility of such transfer, the ability to make ideal measurements in the manner thus far discussed must be rejected.

To illustrate the problem consider three regions, O_1, O_2 and O_3 such that some points in O_1 precede O_2, points in O_2 precede some points in O_3 but all points in O_1 and O_3 are spacelike separated. Further let O_2 be a thickened spacelike hyperplane with O_1 a bounded region to its past and O_3 a bounded region to its future. The associated observables for O_1, O_2 and O_3 will be A, B and C respectively. For a general choice of A, B and C and ρ_0, nonlocality emerges from the fact that measurement of A affects the measurement of C despite them being spacelike separated. As such observers in O_1 and O_3 could prearrange that B is measured thereby allowing a signal to pass between them.

Before presenting an example of this effect in quantum field theory it is worthwhile reviewing the nonrelativistic two outcome Bell measurement on a pair of qubits presented at the end of Chapter 1, cast in the above language. The qubits form a coupled quantum system of three observables: observable A belongs to the first system, C to the second system and B to both. To generalise further a unitary operator resulting from confining Φ on O_1 is used as an arbitrary intervention in this region. A measurement can be seen as a special case of such an intervention by noting that for observable A the measurement converts the density operator ρ into the λ-average $e^{-i\lambda A}\rho e^{i\lambda A}$ with λ a parameter. Thus a measurement of A is facilitated by the operator $U = e^{-i\lambda A}$. Let the coupled systems be two spin 1/2 particles with an initial state $|\downarrow\downarrow\rangle$. The first system is disturbed at time t_1 by applying a unitary operator σ_1 which changes the state to $|\uparrow\downarrow\rangle$. At a later time t_2 B is measured using the the orthogonal projector $(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)/\sqrt{2}$ then at at time t_3 the observable C is measured in the second system. The resulting state in the second system is the pure state $|\downarrow\rangle\langle\downarrow|$ giving an expectation value of $\langle\downarrow| C |\downarrow\rangle$. This differs from the situation where σ_1 was not used in that a totally random density operator $(1/2)(|\uparrow\uparrow\rangle + \langle\downarrow\downarrow|)$ results, leading to an expectation value for C of $(1/2)trC$, thus disturbing the first system has managed to influence the second.

A specific example suffices to demonstrate the effect. Using the interaction picture, a free scalar field $\phi(x)$ initially in its vacuum state is given regions as those described above. The operator U represents a disturbance, which evolves independently of the field, resulting in the initial state ρ_0 transforming as $U\rho_0 U^*$. Explicitly $U = e^{i\lambda\phi(y)}$, where $y \in O_1$, with the observable $C = \phi(x)$, where $x \in O_3$, is measured in O_3. The observable B measured in O_2 can be chosen arbitrarily as its domain of dependence extends across all spacetime. In this case it will be,

$$\mid b\rangle = \alpha \mid 0\rangle + \beta \mid 1\rangle \tag{2.2}$$

where $\mid 0\rangle$ is the vacuum state and $\mid 1\rangle$ is a one particle state, therefore $B =\mid b\rangle\langle b \mid$.

As stated above the region O_1 is disturbed, prompting the state ρ_0 to become $\rho = U\rho_0 U^*$. The probability of $B = 1$ is calculated as $\langle B\rangle = tr\rho_0 U^* B U = trB$ resulting in a normalised state,

$$\sigma = \frac{B\rho B}{trB\rho B} = \frac{B\rho B}{tr\rho B} \tag{2.3}$$

where the projector property, $B^2 = B$, has been used in the second term. Consequently the expectation value for C is,

$$exp(C \mid B = 1) = tr\sigma C = \frac{tr\rho BCB}{tr\rho B} \tag{2.4}$$

Allowing for the outcome that $B = 1$ in the expression $tr\rho B$ gives,

$$exp(C \mid B = 1) = tr\rho BCB = \langle U^* BCBU \rangle \tag{2.5}$$

The probability of achieving the outcome $B = 0$ is $\langle U^*(1 - B)U \rangle$ and by factoring this into 2.4 one acquires,

$$exp(C \mid B = 0) = tr\rho BCB = \langle U^*(1 - B)C(1 - B)U \rangle \tag{2.6}$$

Summing these two results gives the predicted mean vacuum expectation value for C which is,

$$\langle U^* BCBU \rangle + \langle U^*(1 - B)C(1 - B)U \rangle \tag{2.7}$$

For 2.7 to be independent of the magnitude of the disturbance U the derivative of U with respect to λ must vanish for $\lambda = 0$ such that an infinitesimal disturbance has no effect at all. Calculating this derivative one obtains a result of twice the imaginary part of,

$$\langle \phi(y)(C + 2BCB - BC - CB) \rangle \tag{2.8}$$

For locality to be maintained this must be a purely real quantity. The first and last part of this equation are independently real. The former is $\langle \phi(y)\phi(x) \rangle$ which is real because x and y are spacelike separated therefore $\phi(y)$ and $\phi(x)$ commute, while the latter becomes $|\alpha|^2$ when the definition of B is used. Using the notation $\psi(x) \equiv \langle 0 \mid \phi(x) \mid 1 \rangle$ the resulting combination of these terms gives,

$$2(\alpha^* \beta)^2 \psi(x)\psi(y) + (2|\alpha^*|^2 - 1)|\beta|^2 \psi(x)^* \psi(y) \tag{2.9}$$

one can show that this equation is imaginary by allowing $|\alpha|^2 = |\beta|^2 = 1/2$. The second term is eliminated and the remainder has an imaginary factor in α^*. Alternatively one can set $\alpha = 0$ and $\beta = 1$, eliminating the first term and leaving the imaginary factor $\psi^*(x)$. Thus superluminal signalling is an inevitable outcome of making idealised measurements of the general type describe here withe the strength of the signal governed by 2.8.

Chapter 3

Ideal Measurement on Wavepackets

The results of the previous chapter demonstrate that superluminal signalling is an unavoidable consequence of trying to generalise a nonrelativistic idealised measurement to a relativistic setting. Dionigi *et al.* [40] seek to refine the argument by tightening the bounds of applicability to particle states localised in space.

The analysis begins by considering a similar arrangement to that of the previous chapter. The same causal structure of spacetime is adopted wherein region O_1 is an open ball around spacetime point X^μ where $X^0 \leq 0$, region O_2 is a spacalike hypersurface with temporal extent $t = 0$ to $t = T > 0$ and O_3 an open ball around spacetime point Y^μ spacelike separated from X^μ with $Y^0 > T$. Spacetime is $(d+1)$ dimensional Minkowski space with a mostly plus metric inhabited by a free massless scalar field $\hat{\phi}(x)$. As before the results are equally applicable to hyperbolic spacetimes and generalise to free massive fields. An intervention in O_1 is once again achieved by the local unitary operator $e^{i\lambda\hat{\phi}(X)}$. $B = |1\rangle\langle 1|$ forms a measurement in O_2 where $|1\rangle$ is a one particle state. In the interests of absolute clarity the measurement B on O_2 has eigenvalues of 0 and 1 and determines whether a one particle state $|1\rangle$ exists. The basic assumption implicit in eqn. 2.1 is that the measurement B occurs entirely within O_2 and that it is completed by the time $t = T$. A measurement is then performed on $\hat{\phi}(Y)$ in O_3 and the resulting vacuum expectation value, as previously demonstrated in 2.7, is,

$$-Im(\psi(X)^*\psi(Y)) \tag{3.1}$$

which is achieved by setting $\alpha = 0$ and $\beta = 1$.

This superluminal signal is manifestly nonzero for a state $|1\rangle$ with spatial d-momentum $\mathbf{k}$. Furthermore the wavefunction for this specific state conforms to $\psi(Y^\mu + \xi^\mu) = \psi(Y^\mu)$ where ξ^μ is a null vector proportional to the $(d+1)$-momentum $k^\mu = (|\mathbf{k}|, \mathbf{k})$. Hence, the superluminal signal persists independent of the time the measurement of B takes and consequently an ideal measurement of this type is impossible for such a one particle state. One may suspect that this result isn't too improbable as a fixed momentum state is defined across the entirety of a hypersurface so the appearance of nonlocal effects might be expected. To investigate the superluminal signalling outcome more rigorously the above protocol is applied to a spatially localised particle state.

Consider a Gaussian one particle state,

$$| 1_d\rangle := (\pi\sigma^2)^{-\frac{d}{4}} \int d^d k e^{-\frac{(\mathbf{k}-\mathbf{k_0})^2}{2\sigma^2}} a^\dagger_{\mathbf{k}} \mid 0\rangle \tag{3.2}$$

$a^\dagger_{\mathbf{k}}$ is the one particle creation operator, $\mathbf{k}$ represents the one particle d-momentum, $\mathbf{k_0}$ is the mean momentum and σ is the momentum space spread with $|\mathbf{k_0}| >> \sigma$. Initially set $d = 1$ and $k_0^\mu = k_0(1, 1)$ with $k_0 > 0$ so that the wavepacket is moving in a positive spatial direction. For any given null vector $\xi_\mu \propto k_0^\mu$,

$$\begin{aligned}\psi(Y^\mu + \xi^\mu) &= \langle 0 \mid \hat{\phi}(Y^\mu + \xi^\mu) \mid 1_1 \rangle \\ &= (\pi\sigma^2)^{-\frac{1}{4}} \int_{-\infty}^{\infty} \frac{dk}{4\pi|k|} e^{-\frac{(\mathbf{k}-\mathbf{k_0})^2}{2\sigma^2}} e^{ik_\mu(Y^\mu+\xi^\mu)}\end{aligned} \tag{3.3}$$

One can now alter the above wavepacket, in order to avoid the pole at the origin, so that it has support in momentum space for values of $k > \epsilon > 0$ for small values of ϵ. Therefore for momenta contributing to the integral, $k_\mu \xi^\mu = 0$ and this leads to,

$$\psi(Y^\mu + \xi^\mu) = \psi(Y^\mu) \tag{3.4}$$

as for the one particle state earlier. Spacetime coordinate Y can be chosen so that the wavepacket has support on just the positive momenta such that $Y^0 = T$ and $Im(\psi(Y)) \neq 0$. Hence, in 1+1 dimensions where the wavepacket has support in the positive spatial direction it suffers no dissipation and its form is maintained so the superluminal signal remains for any intervention timescale, T. This result is generally applicable for wavepackets with positive momenta support, even nonGaussians. However, for reasons such as infrared divergence, quantum theory in 1+1 dimensional Minkowski space is unphysical and as such attention must turn to 3+1 dimensions.

If the 3-dimensional wavepacket has support in the positive z direction say, but is delocalised across the x and y axes the result is identical to that of the 1-d wavepacket as the superluminal signal is maintained. For a localisation of all spatial dimensions conforming to equation 3.2 where $d = 3$ the magnitude of the wavepacket envelope diminishes due to diffraction into the directions transverse to that of the packet propagation direction. Therefore the superluminal signal strength $Im(\psi(Y))$ decreases as $Y^0 = T$ increases. [40] provides an explicit example of the Gaussian wavepacket $|1_3\rangle$ with a maximum around $\mathbf{k_0} = (0,0,k_0)$ used to calculate $\psi(Y)$ for arbitrary points an the $z-t$ plane with $Y=(t,0,0,z)$. In that analysis $\psi(Y)$ is,

$$\begin{aligned}\psi(Y) &= \langle 0 \mid \hat{\phi}(t,0,0,z) \mid 1_3\rangle \\ &= (\pi\sigma^2)^{-\frac{3}{4}} \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}\sqrt{2|k|}} e^{-\frac{(\mathbf{k}-\mathbf{k_0})^2}{2\sigma^2}} e^{ik_\mu Y^\mu} \\ &= \frac{e^{-k_0^2/2\sigma^2}}{4\pi^{\frac{3}{4}}} \frac{e^{v_-^2/4}D_{-\frac{3}{2}}(v_-) - e^{v_+^2/4}D_{-\frac{3}{2}}(v_+)}{k_0\sigma^{-2}+iz}\end{aligned} \tag{3.5}$$

where $v_\pm = i\sigma[t \pm (z - ik_0/\sigma^2)]$ and $D_v(z)$ is the parabolic cylinder function. This analysis corresponds to decreasing the spatial and increasing the temporal extent T of the region O_2. The results show that the wavepacket envelope does not decay significantly for values of $t = Y^0$ amounting to thickening the temporal dimension of O_2 to that of a few times the extent of the spatial width of the wavepacket. If $z = t+\delta$ such that $0 < \delta < 1/\sigma$ with Y spacelike to X but inside the support of the wavepacket then, for large $t \gg k_0/\sigma^2$ the asymptotic expansion of 3.5 is,

$$Im(\psi(Y)) \ \gamma\sqrt{k_0/\sigma} cos(k_0\delta)t^{-1} \tag{3.6}$$

where γ is of the order $\mathcal{O}(10^{-1})$ meaning that the superluminal signal envelope decays as t^{-1}. This result indicates that at least one of the three interventions, the unitary disturbance U of X, the measurement O_2 between $t=0$ and $t=T$ spanning a time greater than a few times the spatial extent of the wavepacket or the measurement on Y is impossible.

One can demonstrate that an intervention by some external agent producing a nonlocal unitary operation on a field state allows superluminal signalling. Consider the Fock space $\mathcal{F} = \mathcal{H} \oplus \mathcal{H}^\perp$ of the scalar field with $\mathcal{H}^\perp$ the orthogonal complement of $\mathcal{H}$. A 2-dimensional Hilbert space $\mathcal{H}$ spanned by two one particle states $|1\rangle$ and its orthogonal state $|1'\rangle$ forms a subspace of $\mathcal{F}$. Using the three intervention regions O_1, O_2 and O_3 as before the field begins in the state $|\psi\rangle = A|1\rangle + B|1'\rangle$. A unitary operator $e^{i\lambda\hat{\phi}(X)}$ acts at X followed by a unitary operator U enacted in O_2 where,

$$U = e^{i\theta}(C|1\rangle\langle 1| + D|1\rangle\langle 1'| - D^*|1'\rangle\langle 1| + C^*|1'\rangle\langle 1'|) + I^{\perp} \qquad (3.7)$$

where$|C|^2 + |D|^2 = 1$ and $I^{\perp}$ is the identity operator acting on the orthogonal space $\mathcal{H}^{\perp}$. $\hat{\phi}(Y)$ is then measured at Y. The expectation value of $\hat{\phi}(Y)$ is simply,

$$\hat{\phi}(Y) = \langle 1|\psi|e^{-i\lambda\hat{\phi}(X)}U^{\dagger}\hat{\phi}(Y)Ue^{i\lambda\hat{\phi}(X)}|\psi\rangle \qquad (3.8)$$

Following the usual procedure, the value of the superluminal signal is computed by differentiating 3.8 with respect to λ and setting $\lambda = 0$, giving,

$$2Im\langle 1|\psi|\hat{\phi}(X)U^{\dagger}\hat{\phi}(Y)U|\psi\rangle \qquad (3.9)$$

which must be nonzero for the superluminal signal to be nonzero. Setting $A = 1$, $B = 0$, $C = 0$ and $D = -1$ equation 3.8 becomes,

$$\langle 1|\psi|\hat{\phi}(X)U^{\dagger}\hat{\phi}(Y)U|\psi\rangle = \psi'(X)\psi^*(Y) + \psi'(X)\psi^*(Y) \qquad (3.10)$$

where $\psi(\xi) = \langle 0|\hat{\phi}(\xi)|1\rangle$ and $\psi'(\xi) = \langle 0|\hat{\phi}(\xi)|1'\rangle$ are the one particle wavefunctions. For single momentum one particle states residing in a region of length L one can write,

$$\hat{\phi}(X) = L^{-\frac{d}{2}}\sum_{\mathbf{k}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}}}[a_{\mathbf{k}}(X)a_{\mathbf{k}} + a^{\dagger}_{\mathbf{k}}a^*_{\mathbf{k}}(X)] \qquad (3.11)$$

where $u_{\mathbf{k}}(X) = e^{1k_{\mu}X^{\mu}}$. Allowing $|1\rangle = a^{\dagger}_{\mathbf{k}}|0\rangle$ and $|1'\rangle = a^{\dagger}_{\mathbf{k}'}|0\rangle$ 3.9 becomes,

$$\frac{L^{-d}}{\sqrt{2\omega_{\mathbf{k}}\sqrt{2\omega_{\mathbf{k}'}}}}[sin(k'_{\mu}Y^{\mu} - k_{\mu}X^{\mu}) - sin(k_{\mu}Y^{\mu} - k'_{\mu}X^{\mu})] \qquad (3.12)$$

which can be nonzero for spacelike separated X and Y. For particle states $|1\rangle$ and $|1'\rangle$ that are wavepackets, packet spreading becomes important. If X is chosen as in the support of ψ at $t = 0$ and Y in the support of ψ'

at $t = T$ as before, superluminal signalling will occur for values of T a few times greater than the wavepacket width. It appears that causal unitary transformations are those that are completely localised at a spacetime point such as $e^{i\int d^dx f(x)\hat{\phi}(t,\mathbf{x})}$.

Conclusion

Superluminal signalling has been shown to be a persistent outcome of idealised measurement of the kind described here. One possible strategy to avoid such a disaster might be to demand that the measurement regions O_i are arranged such that the coordinates $X \in O_j$ are directly in the casual past of $Y \in O_k$ [39]. This prescription would necessarily exclude measurements being made in many regions not fitting this description. Another strategy might be to impose further caveats on measurement regions by circumventing the need for transitive closure. For instance one could require that for a pair of regions O_j, O_k all pairs of points $x \in O_j$ and $y \in O_k$ be related either by making them spacelike separated or having $x < y$ or vice versa [39]. Not only would this protocol prohibit the measurement detailed in the above two chapters it would also preclude the use of Cauchy hypersurfaces. As before this idea ignores large numbers of regions which do not conform to such strictures.

Alternatively one could maintain the causal structure of measurement regions and instead restrict what observables could be measured by using the impossibility of superluminal signalling. This approach is taken in [35–37, 41]. In the analysis of Chapter 1 this approach informs the kinds of quantum operations possible on bipartite systems with tensor product Hilbert spaces.

It is the case that for a system of bipartite tensor products an idealised measurement on a sum of local field operators complies with causality. For instance $H = H_1 \otimes H_2$ is a tensor product Hilbert space and A and B are self adjoint operators acting on H_1 and H_2 respectively. If the system is in initial state ρ_0 a unitary operation U_1 on H_1 is performed then a measurement of $X = A + B$ is made. Established in Chapter 2, the measurement on X is $e^{i\lambda X} = e^{i\lambda A} e^{i\lambda B}$ as A and B commute. The operation U_1 is followed by applying $e^{i\lambda A}e^{i\lambda B}$, then the partial trace of ρ_0 is taken over H_1. The reduced state for H_2 is not dependent on the operation of U_1. By analogy scalar field theory contains a group of observables which, if amenable to idealised measurements, would similarly not be susceptible to superluminal signalling [40]. This group comprises integrals over spacelike hypersurfaces of local field operators and their conjugate momentum operators [42]. Thus in principle causality would be preserved for the ideal measurement of,

$$a_{\mathbf{k}}(t) + a^{\dagger}_{\mathbf{k}}(t) = \int d^d x \hat{\phi}(t, \mathbf{x}F(\mathbf{x}) + \int d^d x \hat{\pi}(t, \mathbf{x}G(\mathbf{x})) \tag{3.13}$$

where $\hat{\pi} = \dot{\hat{\phi}}$ is the canonical momentum and $F(\mathbf{x})$ and $G(\mathbf{x})$ are Green's functions. Such an observable, though defined across the entire hypersurface t, is composed of a sum of local terms. As such it is 'essentially' local but not 'localised' and of little use in quantum information processing.

A more localised example is that of $b(t) + b^{\dagger}(t)$ where $b^{\dagger}(0)$ is defined as $b^{\dagger}(0)|0\rangle = |1\rangle$ with $|1\rangle$ representing a one particle wavepacket. Allowing $b^{\dagger}(0)$ to be written as,

$$b^{\dagger}(0) = \int d^d k \tilde{\phi}(\mathbf{k}) a^{\dagger}_{\mathbf{k}}(0) \tag{3.14}$$

the one particle wavefunction is then,

$$\psi(t, \mathbf{x}) = \int d^d k (2\omega_{\mathbf{k}})^{-\frac{1}{2}} (2\pi)^{-\frac{d}{2}} e^{i\mathbf{k}\cdot\mathbf{x}} e^{-i\omega_{\mathbf{k}} t} \tilde{\psi}(\mathbf{k}) \tag{3.15}$$

resulting in,

$$b(t) + b^{\dagger}(t) = \int d^d x \left(\hat{\phi}(t, \mathbf{x}) J(\mathbf{x}) + \hat{\pi}(t, \mathbf{x}) K(t, \mathbf{x}) \right) \tag{3.16}$$

where,

$$\begin{aligned} J(\mathbf{x}) &= \int \frac{d^d k}{(2\pi)^{-\frac{d}{2}}} (\frac{\omega_{\mathbf{k}}}{2})^{\frac{1}{2}} \left[e^{i\mathbf{k}\cdot\mathbf{x}} \tilde{\psi}(\mathbf{k}) + c.c \right] \\ K(\mathbf{x}) &= -i \int \frac{d^d k}{(2\pi)^{-\frac{d}{2}} (2\omega_{\mathbf{k}})^{\frac{1}{2}}} \left[e^{i\mathbf{k}\cdot\mathbf{x}} \tilde{\psi}(\mathbf{k}) - c.c \right] \end{aligned} \tag{3.17}$$

where $K(\mathbf{x})$ is twice the imaginary part of $\psi(0, \mathbf{x})$ but $J(\mathbf{x})$ not the real part.

For $b(t) + b^\dagger(t)$ to be localised the value of $\tilde{\psi}(\mathbf{k})$ must give K and J support bounded in space. If $\psi(t, \mathbf{x}) = \psi(0, \mathbf{x})$, K will meet this condition but the further requirement of J to be bounded in support as well makes it unclear whether both can be satisfied. To measure $b(t) + b^\dagger(t)$ one strictly needs to intervene across the whole hypersurface at t but it may be possible to measure in a bounded region like that of 3.2 with the proviso that J and K are exponentially small outside the bound. In principle $b(t) + b^\dagger(t)$ could be measured to good approximation under such constraints but it is debatable as to whether an ideal measurement could be made in practice.

Sorkin [39] also suggests that sums of local operators can prohibit superluminal signalling. He argues that spatially smeared field operators have this character but that operators smeared in time do not. However, [40] makes the assertion that as free scalar field operators at a particular spacetime point are a linear combination of field operators and conjugate momenta at a spacelike hypersurface. Therefore a spacetime smearing of a free field amounts to a spatial smearing of local free field operators. Of course, at the point of measurement the field must interact with the measuring device so in what sense a measured field is free is uncertain.

The ultimate flaw in this analysis may be the treatment of quantum mechanics itself. Dirac has indicated that the Hamiltonian formulation of quantum mechanics is inherently nonrelativistic [43]. The approach based on Lagrangian mechanics is relativistically invariant and leads to the path integral, which supplants Hilbert space as the arena for quantum mechanics in favour of spacetime. This formulation has been advocated by both Hartle [44, 45] and Sorkin [46] in solving foundational problems in quantum theory.

It seems then that for closed quantum systems the path integral approach is most apt for dealing with measurements in relativistic quantum field theory.

Bibliography

[1] M.Planck, Ann. Phys. **4**, 553 (1901)

[2] Shannon, Claude E. A Mathematical theory of Communication, Bell System Technical Journal 27 (3): 379-423, July 1948.

[3] Von Neumann, John , Mathematical Foundations of Quantum Mechanics, Princeton University Press, ISBN 978-0-691-02893-4, 1996.

[4] J.P. Gordon, Proc. IRE 50, 1898-1908, 1962.

[5] D.S. Lebedev, L.B. Levitin, Sov. Phys. Dok. 8, 377, 1963.

[6] A.S. Holevo, Prob. Per. Inf. 9, 3 (1973); Prob. Inf. Trans. (USSR) 9, 110 (1973).

[7] M. Nielsen, B. Schumacher, Phys. Rev. A 54, 2629-2635 (1996)

[8] S. Lloyd, Phys. Rev. A 55, 1613-1622 (1997)

[9] C.H. Bennett, G.Brassard, C. Crepeau, R. Jozsa, A. Peres, W.K. Wootters, Phys. Rev. Lett. 70, 1895-1899 (1993)

[10] J.W. Pan, M. Daniell, S. Gasparoni, G. Weihs, A. Zeilinger, Phys. Rev. Lett. 86, 4435-4438 (2001).

[11] T.C. Zhang, K.W. Goh, C.W. Chou, P. Lodahl, H.J. Kimble, Phys. Rev. A 67, 033802, (2003).

[12] P. Benioff, J. Stat. Phys. 22 , 563-591 (1980)

[13] R.P. Feynman, Int. J. Th. Ph. 21, 467 1982.

[14] D. Deutsch, Proc. Roy. Soc. Lond. A 400, 97117 (1985); Proc. Roy. Soc. Lond. A 425, 7390 (1989).

[15] P. Shor, Proceedings of the 35th Annual Symposium on Foundations of Computer Science pp. 124-134, S. Goldwasser, ed., (IEEE Computer Society, Los Alamitos, CA, 1994).

[16] L.K. Grover, Proceedings, 28th Annual ACM Symposium on the Theory of Computing, p. 212, May 1996.

[17] C. Monroe, D.M. Meekhof, B.E. King, W.M. Itano, D.J. Wineland, Phys. Rev. Lett. 75, 4714, (1995).

[18] Q.A. Turchette, C.J. Hood, W. Lange, H. Mabuchi, H.J. Kimble, Phys. Rev. Lett. 75, 4710, (1995).

[19] C.H. Bennett, G. Brassard, Proceedings of IEEE International Conference on Computers 10-12, 175-179 (1984)

[20] A. Ekert, Phys. Rev. Lett. 67, 661-663 (1991)

[21] M. Czachor, Phys. Rev. A 55, 72 (1997).

[22] A. Peres, P. F. Scudo and D. R. Terno, Phys. Rev. Lett. 88, 230402 (2002)

[23] R. M. Gingrich and C. Adami, Phys. Rev. Lett. 89, 270402 (2002); A. J. Bergou, R. M. Gingrich, and C. Adami, Phys. Rev. A 68, 042102 (2003); J. Pachos and E. Solano, Quant. Inf. Comp. 3 115 (2003).

[24] I. Fuentes-Schuller and R. B. Mann, Phys. Rev. Lett. 95, 120404 (2005).

[25] P. M. Alsing, I. Fuentes-Schuller, R. B. Mann, and T. E. Tessier, Phys. Rev. A 74, 032326 (2006).

[26] G. Adesso, I. Fuentes-Schuller, and M. Ericsson, Phys. Rev. A. 76, 062112 (2007).

[27] G. Adesso and I. Fuentes-Schuller, quant-ph/0702001 to appear in Phys. Rev. D; J. Preskill and D. Gottesman JHEP 0403, 026 (2004); P. Hyden and J. Preskill, JHEP 0709, 120 (2007); S. Lloyd, Phys. Rev. Lett. 96, 061302 (2006); D. Ahn, JHEP 0703, 021 (2007).

[28] D. Terno J. Phys. Conf. Ser. 33, 469-474 (2006); E. Livine and D. Terno, Rev. D 75 084001 (2007); T. Konopka and F. Markopoulou gr-qc/0601028 (2006).

[29] Eduardo Martin-Martinez, Andrej Dragan, Robert B. Mann, Ivette Fuentes, Berry phase quantum thermometer, arXiv:1112.3530v2 [quantu-ph] 28 May 2013

[30] Marvellous Onamu-Kalu, Robert B. Mann and Eduardo Martin-Martinez, Mode invisibility and single photon detection,arXiv:1309.2607v1 [quantu-ph] 10 Sep 2013

[31] Eduardo Martin-Martinez, David Assen and Achim Kempf, Processing quantum information with relativistic motion of atoms, arXiv:1209.4948v2 [quantu-ph] 16 Apr 2013

[32] Eduardo Martin-Martinez, Eric G. Brown, William Donnelly and Achim Kempf, Sustainable entanglement farming from a quantum field, arXiv:1309:1090v1 4 Sep 2013

[33] Jason Doukas, Eric G. Brown, Andrzej Dragan and Robert B. Mann, Entanglement and discord: accelerated observations of local and global modes, arXiv:1209:3461v2 26 March 2013

[34] David Beckman, Daniel Gottesman, M.A. Neilsen and John Preskill, Causal and localisable quantum operations, arXiv:quant-ph/0102043v2, 14 Jun 2001.

[35] Y. Aharanov and D. Z. Albert, States and observables in relativistic quantum field theories, Phys. Rev. D 21, 3316-3324, 1980

[36] Y. Aharanov and D. Z. Albert, Can we make sense out of the measurement process in relativistic quantum mechanics, Phys. Rev. D 24, 359-370, 1981

[37] Y. Aharanov, D. Z. Albert and L. Vaidman, Measurement process in relativistic quantum theory, Phys. Rev. D 34, 1805-1813, 1986

[38] S. Popescu and L. Vaidman, Causality constraints on nonlocal quantum measurements, Phys. Rev. D 49, 4331-4338, 1994

[39] Raphael D. Sorkin, Impossible measurements on quantum fields, arXiv:gr-gc/9302018v2, 20 Feb 1993.

[40] Dionigi M. T. Benincasa, Leron Borsten, Michel Buck, Fay Dowker, Quantum Information Processing and Relativistic Quantum Fields, arXiv:quant-ph/1206.5205 , 22 Jun 2012.

[41] Yakir Aharanov and Daniel Rohrlich, Quantum Paradoxes, Chapter 14, pages 193-209, Wiley-VCH Verlag GmbH, 2008.

[42] Magdelena Zych, Fabio Costa, Johannes Kofler and Caslav Brukner, Entanglement between smeared field operators in the Klien-Gordon vacuum, Phys. Rev., D81:125019, 2010.

[43] Paul A. M. Dirac, The Lagrangian in quantum mechanics, Physikalische Zeitschrift der Sowjetunion, 3(1): 64-72, 1933.

[44] James B. Hartle, The spacetime approach to quantum mechanics. VistasAstron.,37:569, 1993.

[45] JB Hartle, Spacetime quantum mechanics and the quantum mechanics of spacetime, In Gravitation and Quantizations, Session LVII of Les Houches, Volume 1, page 285, 1995.

[46] D. Sorkin, Raphael. Quantum mechanics as measure theory. Mod. Phys. Lett., A9:$3119_{3}128, 1994$